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ALVA'S INSTITUTE OF ENGINEERING & TECHNOLOGY

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BLUE BOOK

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Class: 3rd sem (II year) Div: A

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TEST No.	DATE	MAXIMUM MARKS	MARKS OBTAINED	SIGNATURE STAFF IN CHARGE
I	27-9-16	20	18	27/9/16
II	26-10-16	20	16	27/10/16
III	21-11-16	20	18	28/11/16
		Total	36	28/11/16
		Average	18/20 14/15	28/11/16

Marks in Words: Fourteen only

14/20 15

Signature of Staff in-charge

Signature of HOD

PART-A

1) a).

$$f(x) = x - x^2$$

$$f(-x) = -x - (-x^2) = -x - x^2 = -(x + x^2) \neq f(x)$$

(neither odd nor even).

∴ fourier series

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{\pi} (x - x^2)\,dx$$

$$= \frac{1}{\pi}\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_{-\pi}^{\pi}$$

$$= \frac{1}{\pi}\left[\frac{\pi^2}{2} - \frac{\pi^3}{3} - \left(\frac{(-\pi)^2}{2} - \frac{(-\pi)^3}{3}\right)\right]$$

$$= \frac{1}{\pi}\left[\frac{\pi^2}{2} - \frac{\pi^3}{3} - \frac{\pi^2}{2} - \frac{\pi^3}{3}\right]$$

$$= \frac{1}{\pi}\left[-\frac{2\pi^3}{3}\right] = -\frac{2\pi^2}{3}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x) \cos nx\,dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{\pi} (x - x^2) \cos nx\,dx$$

$$= \frac{1}{\pi}\left[(x-x^2)\frac{\sin nx}{n} - \left((1-2x)\left(-\frac{\cos nx}{n^2}\right)\right) + \left((-2)\left(-\frac{\sin nx}{n^3}\right)\right]_{-\pi}^{\pi}$$

$$= \frac{1}{\pi}\left[(\pi-\pi^2)(0) - \left((1-2\pi)\left(-\frac{\cos n\pi}{n^2}\right)\right) + \left((-2)\left(-\frac{\overset{0}{\sin n\pi}}{n^3}\right)\right)\right] -$$

$$\frac{1}{\pi}\left[(-\pi-(-\pi)^2)\times(0) - \left(\left(1-2(-\pi)\right)\left(-\frac{\cos n\pi}{n^2}\right)\right) + \left((-2)\times 0\right)\right]$$

$$= \frac{1}{\pi}\left[(1-2\pi)\frac{\cos n\pi}{n^2}\right] - \frac{1}{\pi}\left[\left(\cancel{\quad} + (1+2\pi)\frac{\cos n\pi}{n^2}\right]\right.$$

$$\cancel{= \frac{1}{\pi}\left[\frac{\cos n}{n^2}(1-2\pi) - 1 - 2\pi\frac{\cos n\pi}{n^2}\right.}$$

$$= \frac{1}{\pi}\left[\frac{(1-2\pi)\cos n\pi}{n^2} - \frac{(1+2\pi)\cos n\pi}{n^2}\right]$$

$$= \frac{\cos n\pi}{\pi n^2}\left[1-2\pi-1-2\pi\right]$$

$$= \frac{-4\pi\cos n\pi}{\pi n^2} = \frac{-4\pi(-1)^n}{\pi n^2} = -\frac{4(-1)^n}{n^2}.$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx$$

$$= \frac{1}{\pi}\int_{-\pi}^{\pi}(x-x^2)\sin nx\,dx$$

$$= \frac{1}{\pi}\left[(x-x^2)\left(-\frac{\cos nx}{n}\right) - \left(\left(-\frac{\sin nx}{n^2}\times(1-2x)\right)\right) + \left(\frac{\cos nx}{n^3}\times(-2)\right)\right]_{-\pi}^{\pi}$$

$$= \frac{1}{\pi}\left[(\pi-\pi^2)\left(-\frac{\cos n\pi}{n}\right) - 0 - 2\frac{\cos n\pi}{n^3}\right] - \left((-\pi-\pi^2)\left(-\frac{\cos n\pi}{n}\right)\right.$$

$$\left. + 2\frac{\cos n\pi}{n^3}\right]$$

$$= \frac{1}{\pi}\left[(\pi^2-\pi)\frac{\cos n\pi}{n} - \frac{2\cos n\pi}{n^3} + \left((-\pi-\pi^2)\frac{\cos n\pi}{n}\right) + \frac{2\cos n\pi}{n^3}\right]$$

$$= \frac{1}{\pi}\left[\frac{\cos n\pi}{n}\left(\pi^2-\pi-\pi-\pi^2\right)\right]$$

$$= \frac{-2\pi\cos n\pi}{\pi n}$$

$$= \frac{-2(-1)^n}{n}$$

$$\therefore f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

$$= \frac{-2\pi^2}{3/2} + \sum_{n=1}^{\infty}\left(\frac{-4(-1)^n}{n^2}\right)\cos nx + \sum_{n=1}^{\infty}\left(-\frac{2(-1)^n}{n}\right)\sin nx$$

$$= -\frac{\pi^2}{3} - 4\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}\cos nx - 2\sum_{n=1}^{\infty}\frac{(-1)^n}{n}\sin nx$$

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b) $f(x) = \begin{cases} \pi x, & 0 \le x \le 1 \\ \pi(2-x), & 1 \le x \le 2 \end{cases}$ $\quad 2l = 2-0$

$l = 1$

$\phi(x) = \pi x \quad \psi(x) = \pi(2-x)$

$f(2l-x) = f(2-x) = \pi(2-x)$

$= 2\pi - \pi x = \psi(x)$ even

$\therefore b_n = 0.$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi}{l}x + \sum_{n=1}^{\infty} b_n \sin\frac{n\pi}{l}x$$

$$a_0 = \frac{2}{l}\int_0^l f(x)\,dx$$

$$= 2\int_0^1 \pi x\,dx$$

$$= 2\pi\left[\frac{x^2}{2}\right]_0^1 = 2\pi\left[\frac{1}{2}\right] = \pi$$

$$a_n = \frac{2}{l}\int_0^l f(x)\cos\frac{n\pi}{l}x\,dx$$

$$= 2\int_0^1 (\pi x)\cos\frac{n\pi}{1}x\,dx$$

$$= 2\pi\left[x\frac{\sin n\pi x}{n\pi} - \left(\frac{-\cos n\pi x}{(n\pi)^2}\right)\right]_0^1$$

$$= 2\pi\left[0 + \frac{\cos n\pi}{(n\pi)^2} - \left(0 + \frac{1}{(n\pi)^2}\right)\right]$$

$$= \frac{2\pi(-1)^n}{n^2\pi^2} - \frac{2\pi}{n^2\pi^2} = \frac{2}{n^2\pi}(-1)^n - \frac{2}{n^2\pi} \quad (-1)^n - 1$$

$$= \frac{2}{n^2\pi}\left((-1)^n - 1\right)$$

put $n=1$

$$\begin{cases} 0 & \text{if } n \text{ is even} \\ \frac{-4}{n^2\pi} & \text{if } n \text{ is odd} \end{cases}$$

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$\therefore f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi}{l} x + \sum_{n=1}^{\infty} b_n \sin \frac{n\pi}{l} x.$

$f(x) = \pi/2 + \sum_{n=1,3,5}^{\infty} \left(\frac{-4}{n^2\pi}\right) \cos n\pi x + 0.$

put $x = 0$

$f(0) = 0.$

$0 = \pi/2 + \frac{4}{\pi} \sum_{n=1,3,5}^{\infty} \frac{1}{n^2}$

$-\pi/2 = -\frac{4}{\pi}\left[\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots\right]$

$\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} +$

5

2

PART-B

7) i) $3n-4\sin\left(\frac{n\pi}{4}\right)-5a^2$

$u_n = 3n-4\sin\left(\frac{n\pi}{4}\right)-5a^2$

$z(u_n) = 3z(n) - 4z\left[\sin\left(\frac{n\pi}{4}\right)\right] - 5a^2 z[1]$

$z(n) = \frac{z}{(z-1)^2}$

$z[1] = \frac{z}{z-1}$

$z\left[\sin\left(\frac{\pi}{4}n\right)\right]$

$z[\sin n\theta] = \frac{z\sin\theta}{z^2-2z\cos\theta+1}$

$z\left[\sin\left(\frac{\pi}{4}n\right)\right] = \frac{z\sin\pi/4}{z^2-2z\cos\pi/4+1}$

$= \frac{z\times\frac{1}{\sqrt{2}}}{z^2-2z\frac{1}{\sqrt{2}}+1}$

$= \frac{z\frac{1}{\sqrt{2}}}{z^2-\sqrt{2}z+1}$

$\therefore z[u_n] = \frac{3z}{(z-1)^2} - \frac{4\times\frac{1}{\sqrt{2}}z}{z^2-\sqrt{2}z+1} - 5a^2\frac{z}{(z-1)}$

$z[u_n] = \frac{3z}{(z-1)^2} - \frac{2\sqrt{2}z}{z^2-\sqrt{2}z+1} - 5a^2\frac{z}{(z-1)}$

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ii) $u_n = e^{-an}\sin(n\theta)$

$$z[u_n] = z\left[e^{-an}\sin(n\theta)\right] = z\left[(e^{a})^{-n}\sin n\theta\right]$$

$$z[\sin n\theta] = \frac{z\sin\theta}{z^2 - 2z\cos\theta + 1}$$

$$z[a^{-n}u_n] = U[az]$$

$$z\left[(e^{a})^{-n}\sin n\theta\right] = \frac{e^{a}z\sin\theta}{e^{2a}z^{2} - 2e^{a}z\cos\theta + 1}$$

$=$

2

b) $y = ae^{bx}$

apply $\log_e$ both side

$\log_e y = \log_e ae^{bx}$

$\log_e y = \cancel{bx\log}\ \log_e a + \log_e e^{bx}$

$\log_e y = \log_e a + bx\log_e e$

$\log_e y = \log_e a + bx$

$Y = A + bx$

$\therefore\ A = \log_e a \quad Y = \log_e y$

$a = e^{A}$

$\therefore$ normal formula.

$\Sigma Y = nA + b\Sigma x$ $\quad n = 5$

$\Sigma xY = A\Sigma x + b\Sigma x^2$

x	y	$Y = \log_e y$	x^2	xY
77	2.4	0.875	5929	67.375
100	3.4	1.223	10000	122.3
185	7.0	1.945	34225	359.825
239	11.01	2.406	57121	575.034
285	19.6	2.975	81225	847.875

$\Sigma x = 886$ $\quad \Sigma Y = 9.424$ $\quad \Sigma x^2 = 188500$ $\quad \Sigma xY = 1972.409$

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$9.424 = 5A + 886b$

$1972.409 = 886A + 188500b$

$A = 0.183 \quad b = 0.009602$

$a = e^{A}$

$= e^{0.183} = 1.2008.$

$y = ae^{bx}$

$= 1.2008\, e^{0.009602x}$

$=$

5

3). $f(x) = x$ $\quad f(x) = \begin{cases} x & 0 \le x \le \pi \\ 2\pi - x & \pi \le x \le 2\pi \end{cases}$

a)

$\phi(x) = x \qquad \psi(x) = 2\pi - x$

$\phi(2\pi - x) = 2\pi - x$ even

$b_n = 0$

$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$

$a_0 = \frac{2}{\pi} \int_0^{\pi} f(x)\,dx$

$= \frac{2}{\pi} \int_0^{\pi} x = \frac{2}{\pi} \left(\frac{x^2}{2}\right)_0^{\pi} = \frac{2}{\pi} \frac{\pi^2}{2}$

$= \pi$

$a_n = \frac{2}{\pi} \int_0^{\pi} f(x) \cos nx\,dx$

$= \frac{2}{\pi} \int_0^{\pi} x \cos nx\,dx$

$= \frac{2}{\pi} \left[x \frac{\sin nx}{n} - \left(-\frac{\cos nx}{n^2} \right) \right]_0^{\pi}$

$= \frac{2}{\pi} \left[0 + \frac{\cos n\pi}{n} - \left(0 + \frac{1}{n^2} \right) \right]$

$= \frac{2}{\pi} \left[\frac{\cos n\pi}{n} - \frac{1}{n} \right] = \frac{2}{\pi n^2} \left[(-1)^n - 1 \right]$

$\begin{cases} -\frac{4}{n^2 \pi} & \text{if n is odd} \\ 0 & \text{if n is even} \end{cases}$

$\therefore f(x) = \frac{a_0}{2} + \sum_{n=1,3,5}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$

$f(x) = \pi/2 - \frac{4}{\pi} \sum_{n=1,3}^{\infty} \frac{\cos nx}{n^2} + 0.$

put $x = 0.$

$0 = \pi/2 - \frac{4}{\pi} \sum_{n=1,3}^{\infty} \frac{1}{n^2}$

$-\pi/2 = -4/\pi \left[\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots \right]$

$\frac{\pi^2}{8} = \left\{ \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots \right.$

5

$=$

b) $a_0 = \frac{2}{n} \Sigma y$. length of the interval $0 \le x < 6$ $\theta = \frac{n \pi x}{l}$ n

$n = 6$

$a_1 = \frac{2}{n} \Sigma y \cos\theta$ $b_1 = \frac{2}{n} \Sigma y \sin\theta$ $\theta = \pi x$

x	y	$\theta = n\pi/l$ πx	$\sin\theta$	$\cos\theta$	$y\cos\theta$	$y\sin\theta$
0	9	0	0	1	9	0
1	18	π	0	-1	-18	0
2	24	2π	0	1	24	0
3	28	3π	0	-1	-28	0
4	26	4π	0	1	26	0
5	20	5π	0	-1	-20	0

Σ $\Sigma y = 125$

$y\cos\theta = -7$ $\Sigma y \sin\theta = 0$

$a_0 = \frac{2}{n} \Sigma y$

$= \frac{2}{6} \times 125 = 41.66$

$a_1 = \frac{2}{n} \Sigma y \cos\theta$ $b_1 = 0$

$= \frac{2}{6} \times (-7)$

$= -2.33$

$f(x) = 41.66 - 2.33 \cos\theta$

5)a) $u_n = \cos(n\theta)$.

$$\cos n\theta = \frac{e^{i\theta n} + e^{-i\theta n}}{2}$$

$$z[(\cos n\theta)] = \frac{1}{2}\left[z\,[(e^{i\theta})^n + (e^{-i\theta})^n\right]$$

$$= \frac{1}{2}\left[\frac{z}{z-e^{i\theta}} + \frac{z}{z-e^{-i\theta}}\right]$$

$$= \frac{z}{2}\left[\frac{z-e^{-i\theta} + z-e^{i\theta}}{(z-e^{i\theta})(z-e^{-i\theta})}\right]$$

$$= \frac{z}{2}\left[\frac{2z-(e^{i\theta}+e^{-i\theta})}{z^2 - ze^{-i\theta} - ze^{i\theta} + e^{i\theta}e^{-i\theta}}\right]$$

$$= \frac{z}{2}\left[\frac{2z - 2\cos\theta}{z^2 - z(e^{i\theta}+e^{-i\theta})+1}\right]$$

$$= \frac{z}{2}\left[\frac{2z-2\cos\theta}{z^2 - 2z\cos\theta + 1}\right]$$

$$= \frac{2z\,[z-\cos\theta]}{2(z^2-2z\cos\theta+1)} = \frac{z(z-\cos\theta)}{z^2-2z\cos\theta+1}$$

2

(using damping rule $z\,[a^n u_n] = U[z/a]$)

$$z\,[a^n \cos n\theta] = \frac{\frac{z}{a}\left(\frac{z}{a} - \cos\theta\right)}{\left(\frac{z}{a}\right)^2 - 2\frac{z}{a}\cos\theta + 1} = \frac{\frac{z(z-a\cos\theta)}{a^2}}{\frac{z^2-2za\cos\theta+a^2}{a^2}}$$

$$= \frac{z(z-a\cos\theta)}{z^2-2za\cos\theta+a^2}$$

2

b) $x = \sqrt[4]{32}$

$x^4 = 32$

$x^4 - 32 = 0$

$f(x) = x^4 - 32$.

$f(1) = -33$

$f(2) = -16$

$f(3) = 49$

$f(2.3) = -4.0159$

$f(2.4) = 1.1776$.

1.1776

$(2.3, 2.4)$

$$x = \frac{a f(b) - b f(a)}{f(b) - f(a)}$$

$$x_1 = \frac{2.3\,(1.1776) - 2.4\,(-4.0159)}{1.1776 - (-4.0159)} = 2.377$$

$f(x) = 2.377^4 - 32 = -0.076 < 0$

$(2.377, 2.4)$

$$x_2 = \frac{2.377\,(1.1776) - 2.4\,(-0.076)}{(1.1776) - (-0.076)} = 2.378$$

$f(x) = 2.378^4 - 32 = -0.001 < 0$ $(2.378, 2.4)$

$$x_3 = \frac{2.378\,(1.1776) - 2.4\,(-0.001)}{(1.1776) - (-0.001)} = 2.378.$$

$\therefore$ The root of the equation is 2.378

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18/20

36/40

24/9/16

II Internal

PART-A

1) a)

Given $f(x) = \begin{cases} 1 & for\ |x| \leq 1 \\ 0 & for\ |x| > 1 \end{cases}$

$$f(x) = \begin{cases} 1 & -1 \leq x \leq 1 \\ 0 & otherwise \end{cases}$$

Fourier transformation

$$F[f(x)] = \int_{-\infty}^{\infty} f(x)\, e^{iux}\, dx$$

$$= \int_{-\infty}^{\infty} 1 \times e^{iux}\, dx = \int_{-1}^{1} e^{iux}\, dx$$

$$= \left[\frac{e^{iux}}{iu} \right]_{-1}^{1}$$

$$= \frac{e^{iu}}{iu} - \frac{e^{-iu}}{iu}$$

$$= \frac{2}{u}\left[\frac{e^{iu}}{2i} - \frac{e^{-iu}}{2i} \right]$$

$$= \frac{2}{u}\left[\frac{e^{iu} - e^{-iu}}{2i} \right]$$

$$= \frac{2}{u} \sin u = F[u]$$

∴ consider Inverse fourier transformer

$$f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(u)\, e^{-iux}\, du$$

$$f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{2}{u} \sin u\; e^{-iux}\, du$$

Put $x = 0$

$$1 = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{2}{u} \sin u \, e^{-iu(0)} du$$

$$\pi = \int_{-\infty}^{\infty} \frac{1}{u} \sin u \, e^{0} du$$

(using integration property

$$\frac{\sin(-u)}{-u} = \frac{\sin u}{u} \rightarrow \text{even}$$

$$f(x) = 2 \int_0^{\infty} \frac{\sin x}{x} dx)$$

$$\pi = 2 \int_0^{\infty} \frac{1}{u} \sin u \, du$$

(replace u in x by property)

$$\frac{\pi}{2} = \int_0^{\infty} \frac{\sin x}{x} dx$$

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b) $\cancel{F_c[f(x)]}$ $\int_0^\infty f(x)\cos(px)\,dx = \begin{cases} 1-p, & 0 \le p \le 1 \\ 0, & p > 1 \end{cases}$

$$F_c[f(x)] = \int_0^\infty f(x)\cos ux\,dx = F_c[u]$$
$$= \int_0^\infty f(x)\cos px\,dx = F_c[p]$$

using inverse fourier transformer

$$f(x) = \frac{2}{\pi}\int_0^\infty F_c(u)\cos ux\,du$$

$$f(x) = \frac{2}{\pi}\int_0^\infty F_c(p)\cos px\,dp$$

$$= \frac{2}{\pi}\int_0^1 (1-p)\cos px\,dp$$

$$= \cancel{\frac{2}{\pi}}\cancel{\int_0^1}$$

$$= \frac{2}{\pi}\left[(1-p)\frac{\sin px}{x} - (-1)\left(-\frac{\cos px}{x^2}\right)\right]_0^1$$

$$= \frac{2}{\pi}\left[(1-p)\frac{\sin px}{x} - \frac{\cos px}{x^2}\right]_0^1$$

$$= \frac{2}{\pi}\left[-\frac{\cos x}{x^2} - \left(-\frac{1}{x^2}\right)\right]$$

$$= \frac{2}{\pi}\left[\frac{-\cos x}{x^2} + \frac{1}{x^2}\right]$$

$$= \frac{2}{\pi}\left[\frac{1}{x^2}(1-\cos x)\right]$$

$$= \frac{2}{\pi}\left[\frac{1}{x^2}(2\sin^2 x/2)\right]$$

$$f(x) = \frac{4}{\pi x^2}\sin^2 x/2$$

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by using fourier transformer

$$F_c[f(x)] = \int_0^\infty f(x)\cos px\,dx$$

$$= \int_0^\infty \frac{4}{\pi x^2}\sin^2 x/_2 \cos px\,dx$$

put $p=0$.

put $x/_2 = t$.

$dx = dt\,2$

$$= \int_0^\infty \frac{4}{\pi x^2}\sin^2 x/_2\,dx$$

$$= 2\int_0^\infty \frac{4}{\pi (2t)^2}\sin^2 t\,\cancel{dt}$$

$$= 8\int_0^\infty \frac{4}{\pi t^2}$$

$$= 8\int_0^\infty \frac{1}{\pi 4 t^2}\sin^2 t\,dt$$

$$F_c[f(x)] = F_c(p) = \frac{2}{\pi}\int_0^\infty \frac{\sin^2 t}{t^2}\,dt.$$

$$\frac{\pi}{2}F_c(p) = \int_0^\infty \frac{\sin^2 t}{t^2}\,dt$$

-6

$p = 0$?

$F_c(p)$

c) $u_{n+2} + 2u_{n+1} + u_n = n$, $u_1 = 0 = u_0$ $\left(Z(u_n) = U(z) \right)$

$Z[u_{n+2}] + 2Z[u_{n+1}] + Z[u_n] = Z[n]$

$z^2 \left[Z(u_n) - u_0 - u_1 z^{-1} \right] + 2z \left[Z(u_n) - u_0 \right] + Z[u_n] = \dfrac{z}{(z-1)^2}$

$z^2 U(z) - u_0 \times z^2 - z^2 u_1 z^{-1} + 2z U(z) - 2z u_0 + U(z) = \dfrac{z}{(z-1)^2}$

$z^2 U(z) - 0 - 0 + 2z U(z) - 0 + U(z) = \dfrac{z}{(z-1)^2}$

$z^2 U(z) + 2z U(z) + U(z) = \dfrac{z}{(z-1)^2}$

$U(z) \left[z^2 + 2z + 1 \right] = \dfrac{z}{(z-1)^2}$

$U(z) = \dfrac{z}{(z-1)^2 (z+1)^2}$

$z^{-1}\left[\dfrac{z}{(z-1)} \right] = 1$

$z^{-1}\left[\dfrac{z}{(z-1)^2} \right] = n$

$z^{-1}\left[\dfrac{z}{z-(-1)} \right] = (-1)^n$

$z^{-1}\left[\dfrac{z}{(z-(-1))^2} \right] = (+n)(-1)^n$

z^2+2z+1
$z^2+1z+z+1$
$z(z+1)+1(z+1)$
z^2+2z+1
$z^2+1z+1z+1$
$z(z+1)+1(z+1)$
$(z+1)(z+1)$
$(z+1)^2$

$U(z) = \dfrac{z}{(z-1)^2 (z+1)^2} = \dfrac{Az}{(z-1)} + \dfrac{Bz}{(z-1)^2} + \dfrac{Cz}{(z-(-1))} + \dfrac{Dz}{(z-(-1))^2}$ — (1)

$\dfrac{1}{(z-1)^2(z+1)^2} = \dfrac{A}{(z-1)} + \dfrac{B}{(z-1)^2} + \dfrac{C}{(z-(-1))} + \dfrac{D}{(z-(-1))^2}$

$\dfrac{1}{(z-1)^2(z+1)^2} = \dfrac{A(z+1)^2(z-1) + B(z-(-1))^2 + C(z-1)^2(z-(-1)) + D(z-1)^2}{(z-1)^2(z+1)^2}$

$1 = A(z-(-1))^2(z-1) + B(z-(-1))^2 + C(z-1)^2(z-(-1)) + D(z-1)^2$

put $z = 1$

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$1 = \cancel{4A} + B$

$B = 1/4$

put $z = -1$

$1 = -4D$

$D = -1/4$

~~Put~~ comparing coeff of z^3

$0 = A + C \qquad A = +C$

~~comparing coeff of~~ put $z = 0$

$A = 1/2$

$1 = -A + B + C + D$

$1 = -C + B + C + D$

$1 = -2C + B + D$

$1 = -2C + \frac{1}{4} + \frac{1}{4}$

$1 - \frac{1}{2} = 2C \qquad C = -\frac{1}{2}$

$\frac{1}{2} = 2C$

$C = \frac{1}{4}$.

$$\frac{z}{(z-1)^2(z+1)^2} = -\frac{1}{2}\frac{z}{(z-1)} + \frac{1}{4}\frac{z}{(z-1)^2} + \frac{1}{2}\frac{(-z)}{(z-(-1))} + \frac{1}{4}\frac{(-z)}{(z-(-1))^2}$$

$$z^{-1}\left[\frac{z}{(z-1)^2(z+1)^2}\right] = -\frac{1}{2}z^{-1}\left[\frac{+z}{z-1}\right] + \frac{1}{4}z^{-1}\left[\frac{z}{(z-1)^2}\right] + \frac{1}{2}z^{-1}\left[\frac{(-z)}{(z-(-1))}\right] + \frac{1}{4}z^{-1}\left[\frac{(-z)}{z+1}\right]$$

$$u_n = -\frac{1}{2} + \frac{1}{4}n + \frac{1}{2}(-1)^n + \frac{1}{4}(+n)(-1)^n$$

$$u_n = -\frac{1}{2} + \frac{1}{4}n - \frac{1}{2}(-1)^n + \frac{1}{4}(-1)^n n.$$

-2

PART-B

4) c) $y = m_1 x + c_1 \quad y = m_2 x + c_2$

$$\tan\theta = \frac{m_2 - m_1}{1 + m_1 m_2} \quad -① $$

Consider regressions line formula

$$y - \bar{y} = r \frac{\sigma_y}{\sigma_x} (x - \bar{x}) \quad -②$$

$$x - \bar{x} = r \frac{\sigma_x}{\sigma_y} (y - \bar{y})$$

$$(y - \bar{y}) = \frac{1}{r} \frac{\sigma_y}{\sigma_x} (x - \bar{x}) \quad -③$$

$$m_1 = r \frac{\sigma_y}{\sigma_x}, \quad m_2 = \frac{1}{r} \frac{\sigma_y}{\sigma_x}$$

Sub in eq ①

$$\tan\theta = \frac{\frac{1}{r}\frac{\sigma_y}{\sigma_x} - r\frac{\sigma_y}{\sigma_x}}{1 + r\frac{\sigma_y}{\sigma_x} \times \frac{1}{r}\frac{\sigma_y}{\sigma_x}} \quad \sigma_y \cancel{\sigma_x} - r^2 \sigma_y$$

$$= \frac{\frac{\sigma_x \sigma_y - r^2 \sigma_y \sigma_x}{r \sigma_x^2}}{1 + \frac{\sigma_y^2}{\sigma_x^2}} = \frac{\frac{\sigma_x \sigma_y - r^2 \sigma_y \sigma_x}{r \sigma_x^2}}{\frac{\sigma_x^2 + \sigma_y^2}{\sigma_x^2}}$$

$$= \frac{\sigma_y \sigma_x - r^2 \sigma_y \sigma_x}{r(\sigma_x^2 + \sigma_y^2)} = \frac{\sigma_x \sigma_y}{\sigma_x^2 + \sigma_y^2} \left(\frac{1 - r^2}{r} \right)$$

Put $r = 0$

$\tan\theta = \infty$

$\theta = 90^\circ = \frac{\pi}{2}$

$\therefore$ the regressions lines are perpendicular and variable x and y is un-correlated

Put $r = \pm 1$

$\tan\theta = 0$

$\theta = 0$.

$\therefore$ the regression lines are coincide and x and y variable perfect correlation

=

4 b) $3x = \cos x + 1$

$f(x) = 3x - \cos x - 1 = 0$ $\quad f'(x) = 3 + \sin x$

$x_0 = 0.6$

Using newton Raphson method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \left(\frac{3x_n - \cos x_n - 1}{3 + \sin x_n}\right)$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 0.6 - \left(\frac{3x_0 - \cos x_0 - 1}{3 + \sin x_0}\right)$$

-0.0071

$$= 0.6 - \left(\frac{3(0.6) - \cos(0.6) - 1}{3 + \sin(0.6)}\right)$$

$$x_1 = 0.6071$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = x_1 - \left(\frac{3x_1 - \cos x_1 - 1}{3 + \sin x_1}\right)$$

$$= 0.6071 - \left(\frac{3 \times (0.6071) - \cos(0.6071) - 1}{3 + \sin(0.6071)}\right)$$

$$= 0.6071$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 0.6071 - \left(\frac{3 \times (0.6071) - \cos(0.6071) - 1}{3 + \sin(0.6071)}\right)$$

$$= 0.6071$$

6

$\therefore$ The real root of the equation is 0.6071

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4a) $y = ax^2 + bx + c$

$n = 5$

$\Sigma y = a\,\Sigma x^2 + b\,\Sigma x + nc$

$\Sigma xy = a\,\Sigma x^3 + b\,\Sigma x^2 + c\,\Sigma x$

$\Sigma x^2y = a\,\Sigma x^4 + b\,\Sigma x^3 + c\,\Sigma x^2$

x	y	x^2	x^3	x^4	xy	x^2y
1	10	1	1	1	10	10
2	12	4	8	16	24	48
3	13	9	27	81	26	117
4	16	16	64	256	64	256
5	19	25	125	3125	95	475

$\Sigma x = 15$ $\Sigma y = 70$ $\Sigma x^2 = 55$ $\Sigma x^3 = 225$ $\Sigma x^4 = 3479$ $\Sigma xy = 219$ $\Sigma x^2y = 906$

$70 = 55a + 15b + 5c$

$219 = 225a + 55b + 15c$

$906 = 3479a + 225b + 55c$

$a = 0.0326$ $b = 0.7042$ $C = 11.528$

$y = 0.0326x^2 + 0.7042x + 11.528$

4

PART A: 1(a) 6 (b) 6 (c) 3 = 15
B: 4(a) 4 (b) 6 (c) 7 = 17

32/40

16/20

26/10/16

Part - A

1) a) $y = f(x)$

x	y	∇y_0	$\nabla^2 y_0$	$\nabla^3 y_0$	$\nabla^4 y_0$
$x_0 = 40$	$y_0 = 31$				
		42			
$x_1 = 50$	$y_1 = 73$		9		
		51		-25	
$x_2 = 60$	$y_2 = 124$		-16		37
		35		12	
$x_3 = 70$	$y_3 = 159$		-4		
		31			
$x_4 = 80$	$y_4 = 190$				

Using newton forward interpolation formula.

$$y = f(x) = y_0 + \frac{v}{1!}\nabla y_0 + \frac{v(v-1)}{2!}\nabla^2 y_0 + \frac{v(v-1)(v-2)}{3!}\nabla^3 y_0 + \frac{v(v-1)(v-2)(v-3)}{4!}\nabla^4 y_0$$

$v = \frac{x - x_0}{h}$ $\quad h = 10 \quad x = 45$

$$= \frac{45 - 40}{10} = 0.5$$

$$= 31 + \frac{0.5}{1}(42) + \frac{0.5(0.5-1)}{2}(9) + \frac{0.5(0.5-1)(0.5-2)}{6}(-25)$$

$$+ \frac{0.5(0.5-1)(0.5-2)(0.5-3)(37)}{24}$$

$$= 31 + 21 - 1.125 - 1.5625 - 1.4453$$

$f(x) = 47.8672 \simeq 48$ students

$f(45) = 48$ students.

marks b/w 40 and 45 $= f(45) - f(40)$

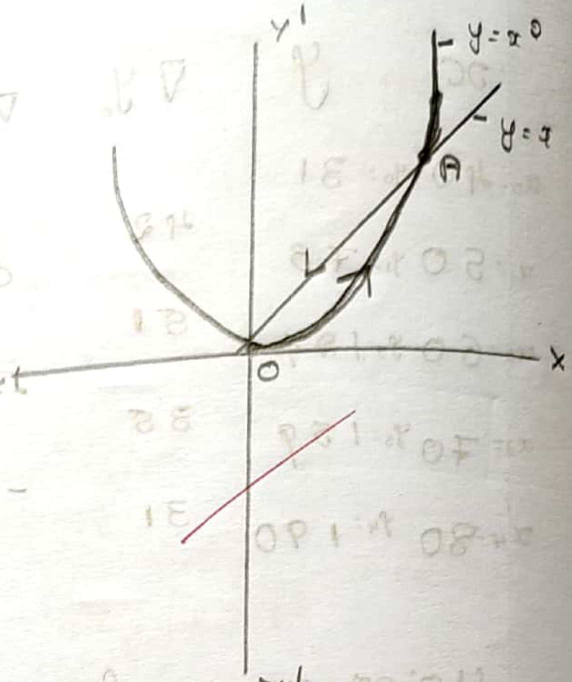
$= 48 - 31$

$= 17$ students

7

c) given $y=x^2$ is parabola

$y=x$ is the straight line.



$y=x^2$

$x^2-x=0$

$x(x-1)=0$

$x=0,1$

$y=x$ when

$y=0$ when $x=0$

$y=1$ when $x=1$

the points $(0,0)$ $(1,1)$

consider Green's theorem

$$\oint_C (m\,dx+n\,dy) = \iint_R \left(\frac{\partial n}{\partial x}-\frac{\partial m}{\partial y}\right)dx\,dy$$

LHS $= \oint_C (m\,dx+n\,dy)$

$m = xy+y^2 \quad n = x^2$

$$\oint_C (m\,dx+n\,dy) = \oint_C \left[(xy+y^2)dx + x^2\,dy\right]$$

I_1, along OA $(0,1)$ $\quad I_1 = \int_O^A \left[(x(x^2)+x^4)\,dx + x^2(2x)\,dx\right]$

$y=x^2$

$dy = 2x\,dx \qquad = \int_0^1 \left[(x^3+x^4)\,dx + 2x^3\,dx\right]$

$$= \int_0^1 \left[(3x^3+x^4)\,dx\right]$$

$$= \left[\frac{3x^4}{4}+\frac{x^5}{5}\right]_0^1 = \cancel{1+\frac{1}{5}} = \frac{3}{4}+\frac{1}{5} = \frac{15+4}{20} = \frac{19}{20}$$

$\cancel{\frac{5+1}{5}} = 1\frac{6}{5}$

I_2, along AO (1,0)

$y = x$

$dy = dx$

$$I_2 = \int_{AO} \left[(x^2 + x^2)\,dx + x^2\,dx \right]$$

$$= \int_1^0 3x^2\,dx$$

$$= \frac{3x^3}{3} \Big|_1^0$$

$$= 0 - 1$$

$$= -1$$

$$I = I_1 + I_2$$

$$= \cancel{6/15} \quad \cancel{\frac{4-5}{5}} \quad \cancel{-\frac{1}{5}}$$

$$= \frac{19}{20} - 1 = \frac{19-20}{20}$$

$$LHS = -\frac{1}{20}$$

$$RHS = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right) dx\,dy$$

$N = x^2$ $\qquad M = xy + y^2$

$\frac{\partial N}{\partial x} = 2x$ $\qquad \frac{\partial M}{\partial y} = x + 2y$

$$\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} = 2x - (x + 2y)$$

$$= x - 2y$$

$$RHS = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right) dx\,dy = \int_{x=0}^{x=1} \int_{y=x}^{y=x^2} (x - 2y)\,dx\,dy$$

$$= \int_{x=0}^{1} \left[xy + \frac{2y^2}{2} \right]_{x}^{x^2} dx$$

$$= \int_0^1 \left[x \times x^2 + \frac{2x^4}{2} - \left(x^2 + \frac{2x^2}{2} \right) \right] dx$$

$$= \int_0^1 \left[x^3 + x^4 - 2x^2 \right] dx$$

$$= \left[\frac{x^4}{4} + \frac{x^5}{5} - \frac{2x^3}{3} \right]_0^1$$

$$= \frac{1}{4} + \frac{1}{5} - \frac{2}{3} - 0$$

$$= \frac{5+4}{20} - \frac{2}{3} = \frac{9}{20} - \frac{2}{3}$$

$$= \frac{27-40}{60}$$

6

$$RHS = \frac{1}{4} - \frac{1}{5} = \frac{5-4}{20} = \frac{-1}{20}$$

$LHS = RHS$

$\therefore$ Hence verified

b) given $y = f(x)$ first order   second order   Third order

x	y
$x_0 = 0$	$y_0 = -4$
$x_1 = 2$	$y_1 = 2$
$x_2 = 3$	$y_2 = 14$
$x_3 = 6$	$y_3 = 158$

$[x_0 x_1] = \frac{y_1 - y_0}{x_1 - x_0} = \frac{2+4}{2-0} = 3$

$[x_1 x_2] = \frac{y_2 - y_1}{x_2 - x_1} = \frac{14-2}{3-2} = 12$

$[x_2 x_3] = \frac{y_3 - y_2}{x_3 - x_2} = \frac{158-14}{6-3} = 48$

$[x_0 x_1 x_2] = \frac{[x_1 x_2] - [x_0 x_1]}{x_2 - x_0} = \frac{12-3}{3-0} = 3$

$[x_1 x_2 x_3] = \frac{[x_2 x_3] - [x_1 x_2]}{x_3 - x_1} = \frac{48-12}{6-2} = 9$

$[x_0 x_1 x_2 x_3] = \frac{[x_1 x_2 x_3] - [x_0 x_1 x_2]}{x_3 - x_0} = \frac{9-3}{6-0} = 1$

Using newton's divided formula.

$f(x) = y_0 + (x - x_0)[x_0 x_1] + (x - x_0)(x - x_1)[x_0 x_1 x_2] + (x - x_0)(x - x_1)(x - x_2)[x_0 x_1 x_2 x_3]$

$= -4 + (x-0)(3) + (x-0)(x-2)(3) + (x-0)(x-2)(x-3)(1)$

$= -4 + 3x + 3[x(x-2)] + x(x-2)(x-3)$

$= -4 + 3x + 3[x^2 - 2x] + x[x^2 - 3x - 2x + 6]$

$= -4 + 3x + 3x^2 - 6x + x^3 - 3x^2 - 2x^2 + 6x$

$= \cancel{x^3 - 3x^2 + 3x^2 - 2x^2 + 3x + 6x - 4}$

$= \cancel{x^3 - 2x^2 + 9x - 4}$

$= 4 + 3x + x^3 - 2x^2$

$= x^3 - 2x^2 + 3x + 4$

$f(4) = 4^3 - 2(4)^2 + 3(4) + 4$

$= 64 - 32 + 12 + 4$

$= 48$

2

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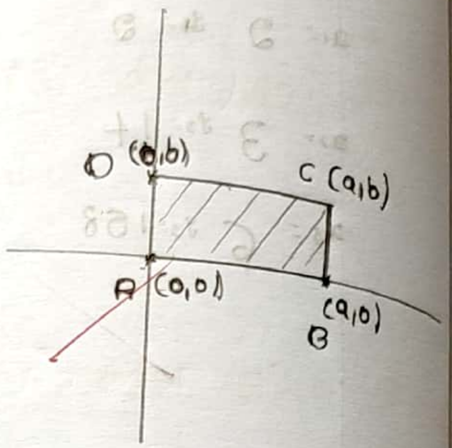
PART-B

3) a). The stoke's theorem is

$$\int_C \vec{F}\cdot d\vec{R} = \iint_R \text{curl}\,\vec{F}\cdot\hat{n}\,ds$$

$$\int_C \vec{F}\cdot d\vec{R} = \iint_R (\nabla\times\vec{F})\cdot\hat{n}\,ds$$



consider LHS $= \int_C \vec{F}\cdot d\vec{R}$

consider AB $\cancel{I_1 = \int_0^a}$

$$= \int_C \left[(x^2+y^2)\hat{i} - 2xy\hat{j}\right]\cdot\left[dx\hat{i} + dy\hat{j} + dz\hat{k}\right]$$

$$\text{LHS} = \int_C \left[(x^2+y^2)\,dx - 2xy\,dy\right]$$

consider AB $y=0,\ dy=0.$ $(0,a)$

$$I_1 = \int_{AB} \vec{F}\cdot d\vec{R}$$

$$= \int_0^a [x^2]\,dx$$

$$= \frac{x^3}{3}\Big|_0^a = \frac{a^3}{3}$$

consider BC , $x=a$ $dx=0$ $(0,b)$

$$I_2 = \int_0^b \left[(a^2+y^2)0 - 2ay\,dy\right]$$

$$= -2a\frac{y^2}{2}\Big|_0^b = -ab^2$$

consider CO , $y = b$ $dy = 0$ $(a, 0)$

$$I_3 = \int_a^0 \left[(x^2 + b^2) dx \right]$$

$$= \frac{x^3}{3} \Big|_a^0 = 0 - \frac{a^3}{3}$$

$$= -a^3/3$$

consider OA , $x = 0$, $dx = 0$ $(b, 0)$

$$I_4 = \int_b^0 \left[\cancel{\quad} \; [y^2(0) - 2(0) \right.$$

$$I_4 = 0$$

$$I = I_1 + I_2 + I_3 + I_4 = a^3/3 - ab^2 - a^3/3 + 0 = -ab^2.$$

$$RHS = \iint_R (\nabla \times \vec{F}) \cdot \hat{n} \, ds$$

$$\nabla \times \vec{F} = \begin{vmatrix} i & j & k \\ \frac{\delta}{\delta x} & \frac{\delta}{\delta y} & \frac{\delta}{\delta z} \\ x^2 y^2 & -2xy & 0 \end{vmatrix} = i(0) - j(0) + k\left(\frac{\delta}{\delta x}(-2xy) - \frac{\partial}{\partial y}(x^2 y^2)\right)$$

$$= k(-2y - 2y) = -4yk.$$

$(\nabla \times \vec{F}) \cdot \hat{n} \cdot ds$ $\hat{R} = dydz\hat{i} + dzdx\hat{j} + dxdy\hat{k}$ $\hat{n} \cdot ds = (dydz\hat{i} + dzdx\hat{j} + dxdy\hat{k}) \cdot (-4y\hat{k})$

$$\hat{n} \, ds = -4y \, dx \, dy$$

$$= \iint_R (-4y) \, dx \, dy$$

$$= \int_{x=0}^{a} \int_{y=0}^{b} (-4y) \, dx \, dy$$

$$= \int_0^a \left(-4\frac{y^2}{2}\right)\Big|_0^b dx = \int_0^a (-2b^2) \, dx$$

$$RHS = -2xb^2 \Big|_0^a = -2ab^2$$

6

7

$LHS \neq RHS$

Hence not Proved

b) given $\vec{F} = 3xy\hat{i} - y^2\hat{j}$

The line integral $= \int_C \vec{F} \cdot d\vec{R}$ $\quad \vec{R} = x\hat{i} + y\hat{j} + z\hat{k}$
$d\vec{R} = dx\hat{i} + dy\hat{j} + dz\hat{k}$

$$= \int_C (3xy\hat{i} - y^2\hat{j}) \cdot (dx\hat{i} + dy\hat{j} + dz\hat{k})$$

$$= \int_C (3xy\,dx - y^2\,dy)$$

given $y = 2x^2$ $\quad (0,0)$ to $(1,2)$

$dy = 2(2x)\,dx$ the ~~xyz~~ consider x plane the limit from

$= 4x\,dx$. $\quad$ 0 to 1

$$= \int_0^1 \left(3x(2x^2)\,dx - (2x^2)^2(4x)\,dx\right)$$

$$= \int_0^1 \left(6x^3\,dx - 16x^5\,dx\right)$$

$$= \left[\frac{6x^4}{4} - 16\frac{x^6}{6}\right]_0^1$$

$$= \frac{6}{4_2} - \frac{16^8}{6_3}$$

$$= \frac{3}{2} - \frac{8}{3}$$

$$= \frac{9-16}{6} = -7/6$$

6

c)

x	y	first order	second order	third order
$x_0 = 0$	$y_0 = 1$			
		1		
$x_1 = 1$	$y_1 = 2$		-2	
		-1		12
$x_2 = 2$	$y_2 = 1$		10	
		9		
$x_3 = 3$	$y_3 = 10$			

Consider newtons forward interpolation formula.

$$y = y_0 + \frac{v}{1!}\nabla y_0 + \frac{v(v-1)}{2!}\nabla^2 y_0 + \frac{v(v-1)(v-2)}{3!}\nabla^3 y_0$$

$h=1,$

$$v = \frac{x-x_0}{h} = \frac{x-0}{1} = x.$$

$$= 1 + x(1) + \frac{x(x-1)}{2}(-2) + \frac{x(x-1)(x-2)}{6}(12).$$

$$= 1 + x - [x(x-1)] + 2x(x-1)(x-2)$$

$$= 1 + x - (x^2 - x) + 2x[x^2 - 2x - x + 2]$$

$$= 1 + x - x^2 + x + 2x^3 - 4x^2 - 2x^2 + 4x$$

$$= 2x^3 - x^2 - 4x^2 - 2x^2 + x + x + 4x + 1$$

$$f(x) = 2x^3 - 7x^2 + 6x + 1$$

7

$$f(4) = 2(4)^3 - 7(4)^2 + 6(4) + 1$$

$$= 128 - 112 + 24 + 1$$

$$= 41$$

Part A: 16
B: 19

35/40

18/20

22/11/16