PART-A

Module 1

Hydrology

unit 1 : Introduction and Precipitation.

→ The word hydrology is derived from the Greek words, hydro, which means water, & logos, which means Science. Thus hydrology is the science which is concerned with all waters on Earth, its occurence, distribution & circulation its physical & chemical properties, its effects on the Environment & on life of all forms.

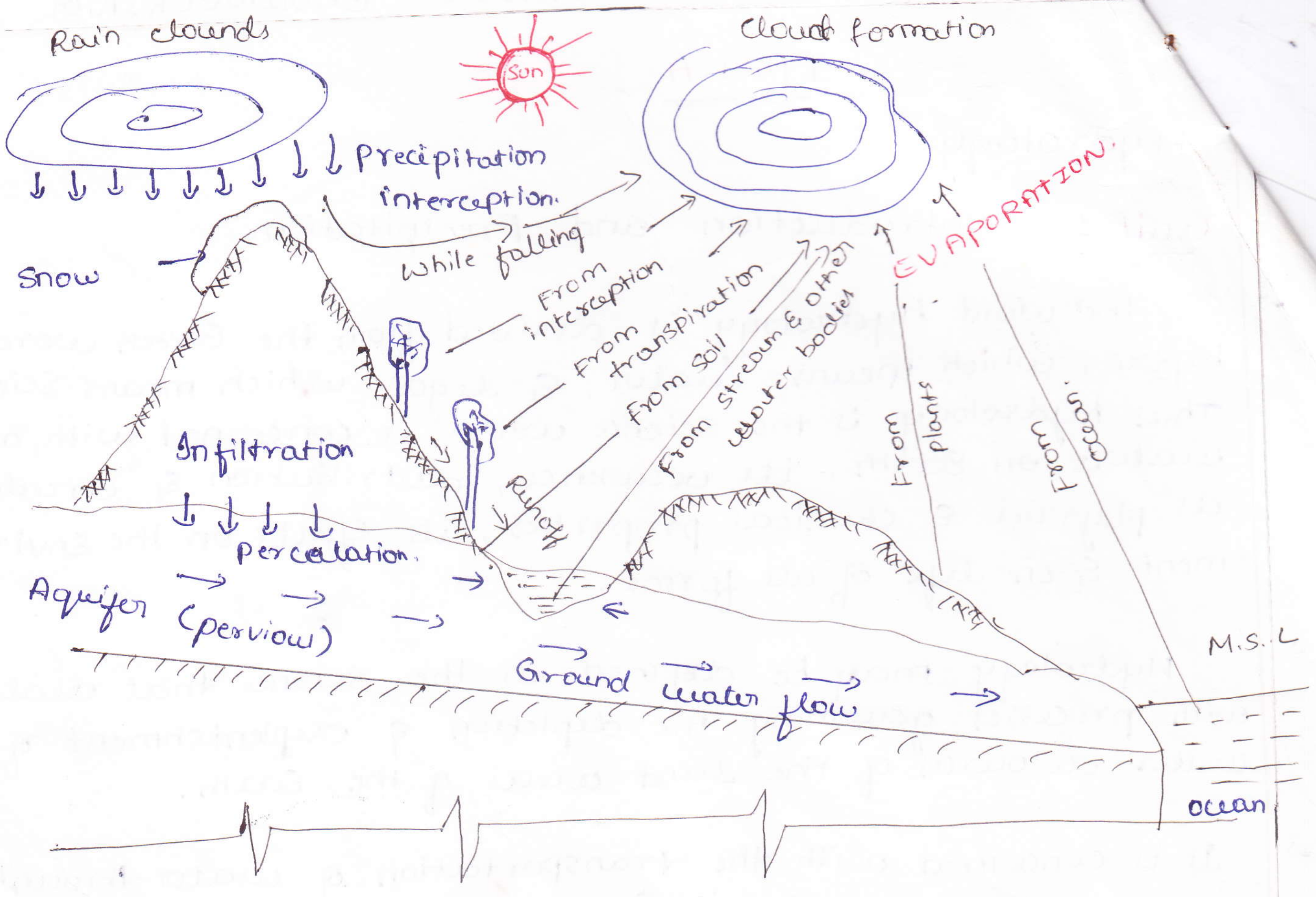
→ Hydrology may be defined as the science that deals with processes governing the depletion & replenishment of water resources of the land areas of the Earth.

→ It is concerned with the transportation of water through the air, over the ground Surface & through the strata of the Earth. It is Science that treats of the various phases of the hydrologic cycle.

→ Hydrology finds many application in Hydraulics, Agricultural water resources & other branches of Engineering. Therefore, the names like Engineering Hydrology & Applied Hydrology are also often used.

Hydrologic cycle

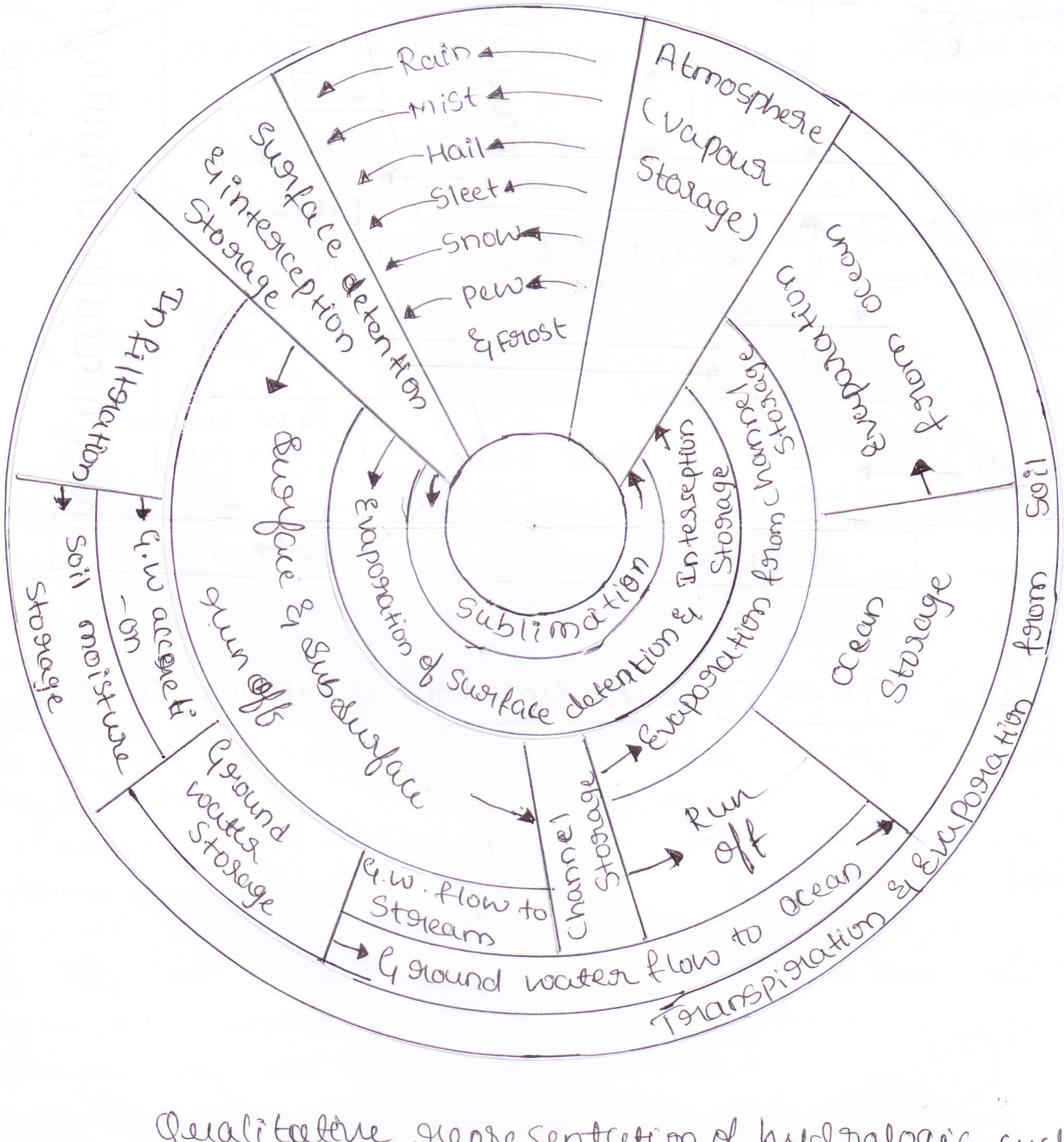
Water Exists on planet Earth in its three forms viz. gaseous, liquid & solid forms & is circulated mainly by Solar & planetary forces. The sun provides the Energy for the evaporation of Sea water & Earth's gravitational field & coriolis force contributes to the circulation of water. The group of numerous arcs which represent the different paths through which the water in nature circulates & is transformed is what is known as hydrologic cycle.



Descriptive representation of hydrologic cycle

These ares penetrates into the three parts of the total Earth System, namely, Atmosphere, Hydrosphere & Lithosphere. The atmosphere is the gaseous envelope above the hydrosphere. Hydrosphere is the bodies of water that cover the surface of the Earth. Lithosphere is the solid rock below the hydrosphere. The activities of water extend through these three parts of the Earth System from an average depth of about 1 km. in the lithosphere to a height of about 15 km in the atmosphere.

The hydrologic cycle has no beginning or end as the water in nature is continuously kept in cyclic motion. However, for the purpose of description the cycle may be visualised to commence with the precipitation from the atmosphere. precipitation may take place in liquid form as rain & also in solid form as hail, snow, dew, frost etc.



Qualitative representation of hydrologic cycle

(Horton's cycles)

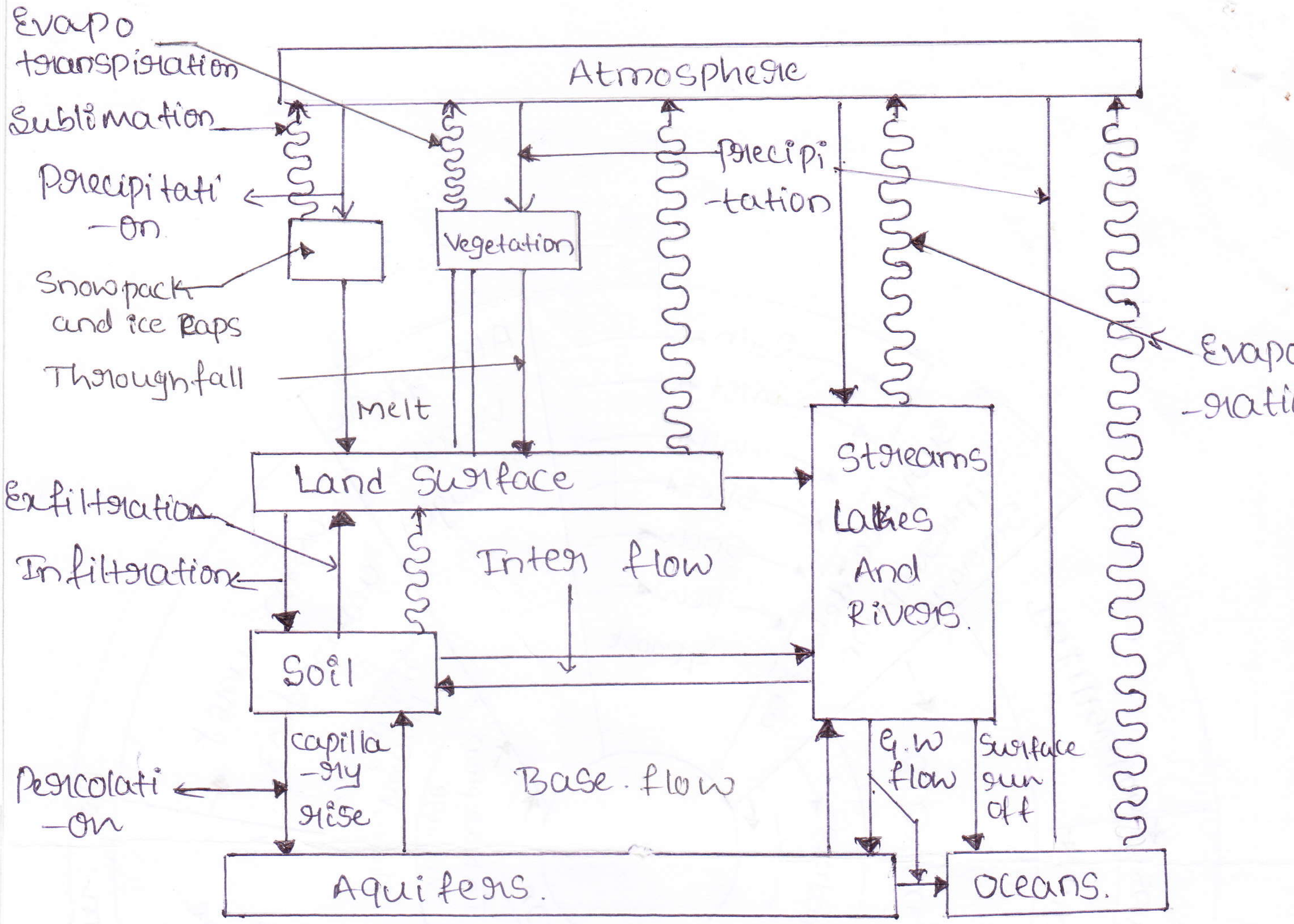


fig Engineering representation of hydrologic cycle

While precipitation is taking place, a part of it may evaporate & reach back the atmosphere. Some more precipitation is intercepted by the trees & vegetation & the rest of it only would reach the ground. The intercepted precipitation eventually evaporates into the atmosphere. The precipitation reaching the ground surface is called the throughfall. Considerable portion of the throughfall gets infiltrated into the ground & that in excess of infiltration would be detained temporarily on the ground before it becomes overland flow & subsequently surface runoff. The precipitation falling directly over the streams is called the channel precipitation & it readily becomes runoff without any delay.

The infiltrated water may be distributed in different ways. First, it supplies moisture to the vegetation & after utilising it for the sustenance of their life, the vegetation sends this moisture back into the atmosphere through the leaves by a process known as transpiration. Secondly, the infiltrated water may percolate deep & become ground water supply to surface streams known as the ground water runoff, or it may become ground water supply to surface to oceans. The groundwater runoff is sometimes referred to as the baseflow or the interflow. The total streamflow which is the sum of the surface runoff & the groundwater runoff ultimately joins the oceans wherefrom it again evaporates into the atmosphere thus completing the hydrologic cycle. The entire cycle repeats when the atmosphere moisture precipitates on to the ground after cloud formation. Thus the hydrologic cycle consists of various complicated processes such as precipitation, interception, evaporation & transpiration, infiltration, percolation, storage & runoff.

Water-budget method & India's water budget:

Water-budget method is used to determine the Evaporation losses from a reservoir or a river, etc. is by the use of Storage Equation, which is given by the following Equation.

$$P + Q_i \pm Q_u = E + Q_o \pm \Delta Q_s$$

Where,

P = Total precipitation on the water Surface

Q_i = Total Surface inflow

Q_u = Total underground inflow or outflow (+ve value for inflow, & −ve value for outflow)

E = Evaporation from the water surface

Q_o = Surface outflow.

ΔQ_s = change in Storage (+ve for increase in Storage, −ve for decrease)

All these factors are converted into the same units, generally expressed as the depth on the water area for some convenient time interval. If all other terms are known, E can be calculated.

Definition of precipitation:

→ water evaporates from water surfaces like streams, rivers, oceans, ponds & also from the land & plants, in the form of water vapour. These water vapour get collected in the atmosphere & behave like a gas.

→ under a normal range of temperature & pressure, the water vapour obeys the various gas law (i.e., Boyle's law, charle's law etc). As the evaporation continues, the amount of atmospheric vapour goes on increasing. But since a space can hold only a certain fixed amount of water vapour in the presence of a solid or a liquid surface, a state is reached when any further addition of vapour will get condensed on the surfaces. The vapour may get condensed in different forms, such as mist, rain, hail, snow, sleet etc. The evaporated water thus returns to the earth surface in any of these forms.

→ The water which comes back to the surface of the earth in its various forms, like rain, snow, hail, sleet etc is known as precipitation.

Forms of precipitation:

Drizzle: It is fine sprinkle of very small & rather uniform water drops with diameters between 0.1 & 0.5 mm. The drops are so small that they seem to float in air. To qualify as drizzle (also called mist), the drops must not only be small but they must also be very numerous. The intensity of drizzle rarely exceeds 1 mm/h.

Rain: Rain is the precipitation of liquid water in which the drops are generally larger than 0.5mm in size.

Glaze: The ice coating formed when rain or drizzle freezes as it comes in contact with cold objects at the ground is called the glaze.

Sleet: when rain drops are frozen while falling through a layer of subfreezing air (below 0°c) near the Earth's surface or refreezing of largely melted ice crystals occurs, transparent globular grains of ice known as sleet or ice pellets are formed. The pellets are generally between 1mm & 4mm in diameter.

Snow: precipitation in the form of ice crystals is called snow. when the ice crystals fuse together, it is called snowflake.

Hail: precipitation in the form of balls or irregular lumps of ice over 5mm in diameter is called hail. Hail stones are generally composed of alternating ice & opaque snow like layers as a result of repeated ascents & descents within the cloud during their formation. Hail occurs almost exclusively in violent & prolonged thunderstorms.

Dew: Dew forms directly by condensation on the ground mainly during the night when the surface has been cooled by the outgoing radiation.

Types of precipitation:

Although the moisture is always present in the atmosphere, but it is condensed only when the air is cooled, so that it becomes saturated with the same amount of water vapour. The usual mechanism by which the air is cooled to cause precipitation is the lifting of the air mass. There are three different method by which the air mass gets lifted, so as to cause cooling & condensation of the atmospheric water vapour, & the subsequent precipition mainly in the form of rain or sometimes under special conditions as snow, hail, sleet etc. Depending upon the way in which the air is cooled, as to cause precipitation, we can have three kinds of precipitation, as given below.

Cyclonic precipitation:

It is caused by the lifting of an air mass due to the pressure difference. If low pressure occurs in an area, air will flow horizontally from the surrounding area, causing the air in the low pressure area to lift. The precipitation that results is called non-frontal cyclonic precipitation. If one air mass lifts over another air mass, the precipitation is called frontal cyclonic precipitation. The boundary between these two air masses of different temperatures & densities (one warm air mass & other colder) is known as a front or frontal surface. The large whirling mass of air, at the centre of which the baromet(atm)-ric pressure is low, is known as a cyclone.

The air that rushes horizontally into the low pressure area changes into a whirling mass because of the rotary motion of the Earth about its own axis. The cyclone is a very large mass of air ranging from 800 to 1600Km in diameter & moving with a velocity of about 50km/hr.

The central portion of this cyclone, where the pressure is low, acts like a chimney, through which the air gets lifted, expands, cools & finally gets condensed causing precipitation.

Cyclonic precipitation can occur in the form of drizzle, intermittent rain, of steady rain. If the precipitation is caused by a cold front, it is very intense & of short duration; while that caused by a warm front is more continuous. Third possibility is that of an occluded front. An occluded front occurs when a cold front over-takes a warm front. The precipitation pattern is a combination of both cold & warm frontal distributions. A cold front is that in which the warm air replaces the colder air, whereas in a warm front the case is opposite.

Cyclones are responsible for most of the rains in the central part of the united States, & for most of the winter rains in Haryana & Punjab in India.

Convective precipitation:

It is caused due to the upward movement of the air that is warmer than its surroundings. Generally this kind of precipitation occurs in tropics, where on a hot day, the ground surface gets heated unequally, causing the warmer air to lift up; as the colder air comes to take its place. The vertical air currents develop tremendous velocities & are hazardous to aircraft. precipitation occurs in the form of showers of high intensity & short duration.

Orographic precipitation:

It is the most important precipitation & is responsible for most of the heavy rains in India. orographic precipitation is caused by air masses which strikes some natural topographic barriers like mountains, & cannot move forward & hence rise up, causing condensation & precipitation. The greatest amount of pricipitation fall on the windward side, & the leeward side often has very little precipitation

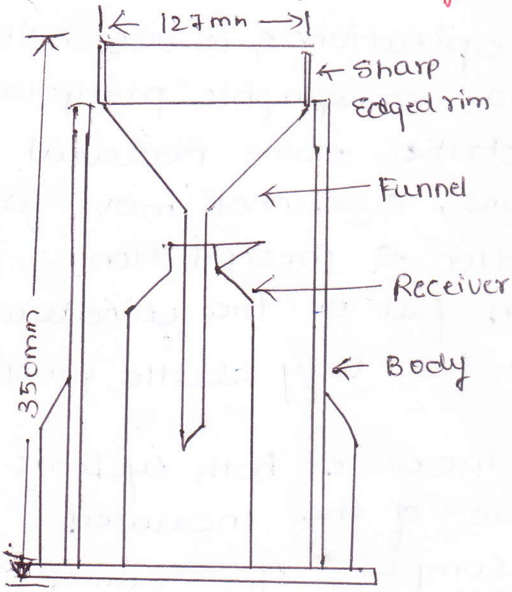
Orographic barriers tend to increase both cyclonic & orographic precipitation because of the increased lifting involved. The rainfall is composed of showers & steady rainfall. A striking example of this kind of natural barriers is the Southern slope of Himalayas in India. The winds, heavily laiden with moisture from the Bay of Bengal, strike the Southren slope of Himalayas, causing intense rains, so much so that in cherrapunji, the average annual rainfall is of the order of 1270cm

Measurement of precipitation:

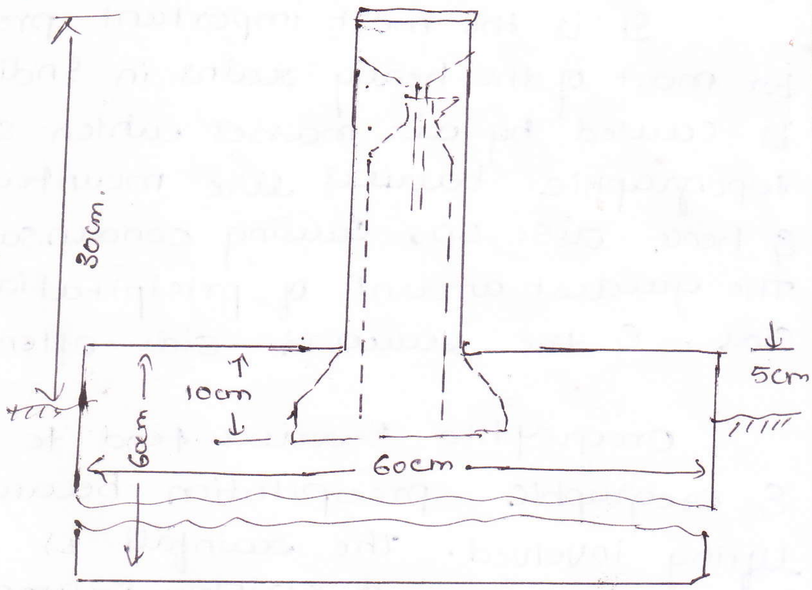
Precipitation is usually measured in millimetres & tenths of millimetres. If it is less than 1mm it is recorded as trace. Rainfall with an intensity of 2.5mm/h is called light rain, between 2.5mm/h & 7.5 mm/h it is termed as moderate rain & when it exceeds 7.5mm/h it is termed as heavy rain. A small surface area is taken for the purpose of measurement & the volume of precipitation water collect over that area is divided by the area to give the depth of precipitation.

The precipitation is measured by an instrument called raingauge. Raingauge is also variously known as hyetometer, ombrometer or pulvimeter. The raingauges are of two types: Non-recording type or ordinary raingauge
Recording type or automatic raingauges.

Non-Recording Raingauges (Symon's raingauge)



Symon's raingauge



Installation of non-recording raingauge

Standard non-recording raingauge prescribed by IMD is the Symon's gauge the details of which are shown in above fig. The gauge consists of a funnel with a sharp Edged rim of 127mm diameter, a cylindrical body, a receiver with a narrow neck & handle & a splayed base which is fixed in the ground. The receiver should have a narrow neck & should be sufficiently protected from radiation to minimise the loss of water from the receiver by Evaporation. To prevent rain from splashing in & out, the vertical wall of the Sharp Edged rim is made deep enough & the Slope of funnel steep enough (at least 45°). The rain falling into the funnel is collected in the receiver kept inside the body & is measured by means of a special measure glass (supplied along with the gauge) which is graduated in mm. The receiver has a capacity of 175mm of rain. In regions of heavy rainfall, raingauges with receivers of 375mm or 1000mm capacity may be used.

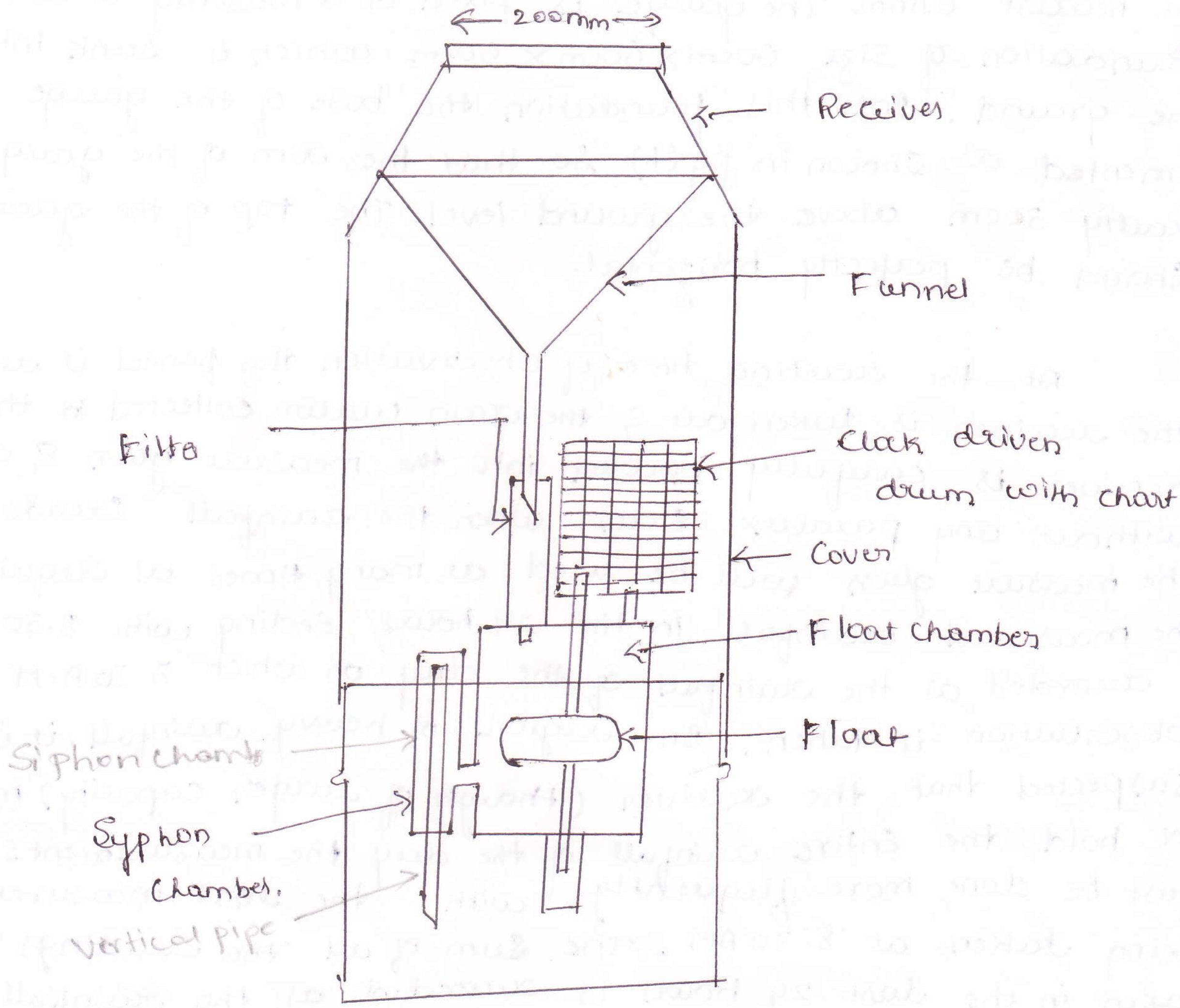
The measure glass has a capacity of 25mm & can be read to nearest 0.1mm. The gauge is fixed on a masonry of concrete foundation of size 60cm x 60cm x 60cm which is sunk into the ground. Into this foundation the base of the gauge is cemented as shown in fig (b) so that the rim of the gauge is exactly 30cm above the ground level. The top of the gauge should be perfectly horizontal.

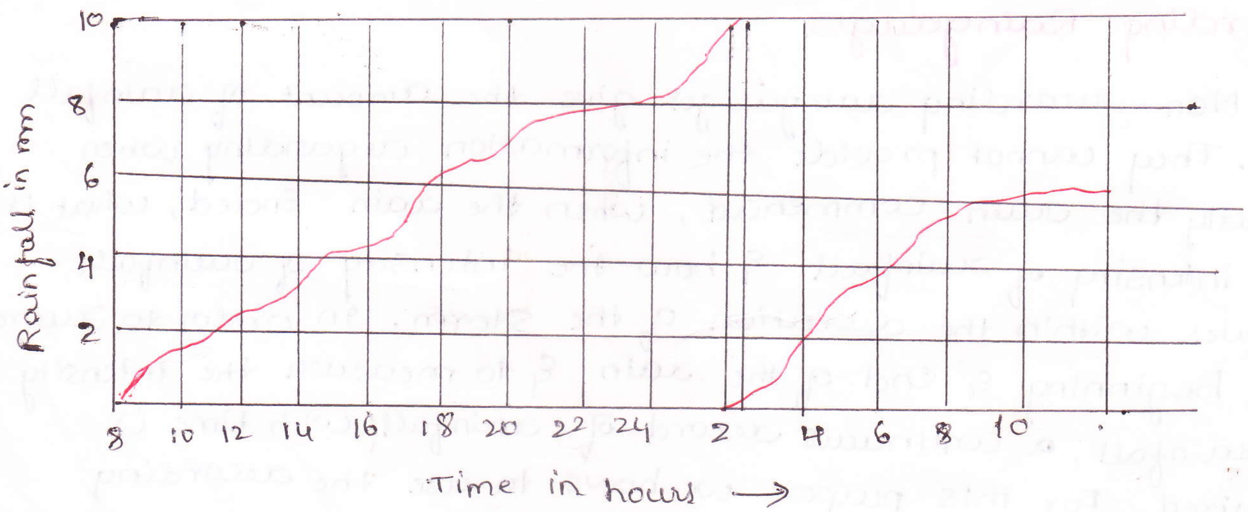
At the routine time of observation the funnel is removed, the receiver is taken out & the rain water collected in the receiver is carefully poured into the measure glass & read without any parallex error. When the rainfall exceeds 25mm the measure glass will be used as many times as required. The measured rainfall in the 24 hours ending with 8.30 A.M. is recorded as the rainfall of the day on which 8.30 A.M observation is taken. In regions of heavy rainfall, if it is suspected that the receiver (though of larger capacity) may not hold the entire rainfall of the day the measurements must be done more frequently with the last measurement being taken at 8.30 AM. The sum of all the readings taken in the last 24 hours is recorded as the rainfall of that day.

Recording Raingauges

Non-recording raingauges give the amount of rainfall only. They cannot provide the information regarding when exactly the rain commenced, when the rain ended, what is the intensity of rainfall & how the intensity of rainfall varies within the duration of the storm. In order to record the beginning & end of the rain & to measure the intensity of rainfall, a continuous record of rainfall with time is required. For this purpose we have to use the recording raingauges. Recording raingauges usually work by having a clock-driven drum carrying a graph on which a pen records the cumulative depth of rainfall continuously.

Float type raingauge or (Siphon type raingauge)





Rainfall chart from float type raingauge.

This type of raingauge is also known as the Siphon type raingauge as it uses the siphon mechanism to Empty the rainwater collected in the float chamber. This is adopted by I.M.D. The details of construction of this type of raingauge are shown in fig(a). Rainwater entering the gauge at the top is led into the float chamber through a funnel & filter. The purpose of the filter is to prevent dust & other particles from entering the float chamber which may hinder the Siphon mechanism.

The float chamber consist of a float with a vertical Stem protruding outside, to the top of which a pen is mounted. This pen rest on a chart secured around a clock driven drum. There is a Small Compartment by the side of the float chamber which is connected to the float chamber through a Small opening at the bottom. This is called the siphon chamber which houses a small vertical pipe with bottom End open & the top End almost touching the top of the chamber.

During the storm the rainwater collected in the float chamber raises the water surface in it & along with the water surface in it & along with the water surface the float also rises enabling the pen to make a trace of cumulative depth of rainfall on the chart. When the float chamber is completely filled with water, the pen reaches the top of the chart. At this instant the siphoning occurs automatically through the pipe in the siphon chamber, the float chamber is emptied & the pen is brought to zero on the chart again. As the rain continues the pen rises again from the zero of the chart. The complete Siphoning should be over in less than 15 seconds of time. This gauge cannot record precipitation in the form other than rain unless some sort of heating device is provided inside the gauge. The float may be damaged if the rainfall catch freezes.

A chart from a float type raingauge with siphoning taking place during the storm is shown in fig (b). This chart indicates that the gauge has siphoned once at 1:30h of the next day. Thus, the cumulative depth of rainfall recorded by the gauge at 5h of the next day, for Example, is 10+4=14mm. If the rainfall is of large intensity, the siphoning may occur more than once during the period of the chart.

Location of Raingauge

The amount of rainfall collected by a raingauge depends on its Exposure conditions & therefore great care must be Exercised in Selecting a Suitable Site for its location. According to Indian Standards the following precautions must be strictly observed while selecting a site for a raingauge.

i) The gauge shall be placed on a level ground, not upon a slope or a terrace & never on a wall or roof.

ii) On no account the raingauge shall be placed on a slope such that the ground falls away steeply in the direction of prevailing wind.

iii) The distance of the raingauge from any object shall not be less than twice the height of the object above the rim of the gauge.

iv) Great care shall be taken at mountain & coast stations so that the gauges are not unduly exposed to the sweep of the wind. A belt of trees or a wall on the side of the prevailing wind at a distances Exceeding twice its height shall form an efficient shelter.

v) In hills where it is difficult to find a level space, the site for the gauge shall be chosen where it is best shielded from high winds & where the wind does not cause Eddies.

The location of the gauge should not be changed without taking suitable precations.

Average Annual Rainfall & Index of wetness

The amount of rain collected by a given rain gauge in 24 hours is known as daily rainfall, & the amount collected in one year is known as annual rainfall. This annual rainfall at a given station should be recorded over a number of years, say 35 to 40 years or so. In India, this rainfall cycle period is taken to be about 35 years. The mean of the annual rainfalls over a period of 35 years or so, is therefore, known as average annual rainfall or normal annual rainfall of the given station.

Example, when we speak that cherrapunji gets 1270 cm of yearly rain, it means that this figure has been averaged over a long period of about 35 years. This is known as average rainfall. But in any given particular year, the rain may not fall equal to this amount. It may be less than this average value or may exceed this average value. The ratio of the actual rainfall in a given particular year at a given place to the normal rainfall of that place, is known as index of wetness

$$\text{Index of wetness} = \frac{\text{Actual rainfall in a given year at a given place}}{\text{Normal Annual rainfall of that place.}}$$

Index of wetness, thus, gives us an idea of the wetness of the year, & hence, it indicates the deficiency of rain.

60% index of wetness means a rain deficiency of 40%.

30-40% → large deficiency

45-60% → Serious deficiency

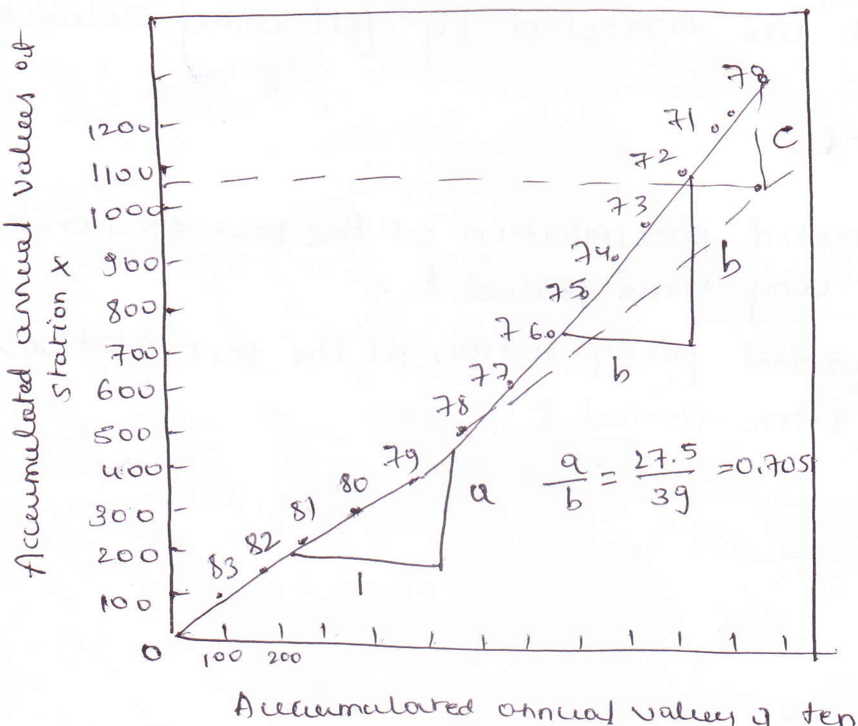
> 60% → disastrous deficiency

If A.A.R < 40 cm → Arid climate, 40-75 cm - Semi-arid climate

> 75 cm - humid climate.

The year in which the rainfall is less than the average annual rainfall, is called a bad or a sub-normal year, & if the rainfall in a given year is more than the average, then it is known as good year. If the rainfall in a particular year is approximately equal to the annual average value, then it is known as a normal year or an average year.

Double Mass curve of rainfall



Double mass curve is a technique used for checking the consistency of a record. The trend in the rainfall records at a station may slightly change after some years due to change in the environment of a station, or tampering of the instrument or shift in the observation practices.

A double mass curve is a graph plotted between the accumulated annual rainfall at a given Station X (test stan) versus accumulated annual values of the average of group of base station, for various consecutive time periods, as shown in fig.
A break in the slope of the resulting plot indicates a change in the precipitation regime of station X.
The precipitation values at the Station X, beyond the period of break are corrected by following relationship

$$P_{xc} = P_x \cdot \frac{a}{b}$$

P_{xc} = corrected precipitation at the test station X, at any time period t.

P_x = recorded precipitation at the test station X at time period t.

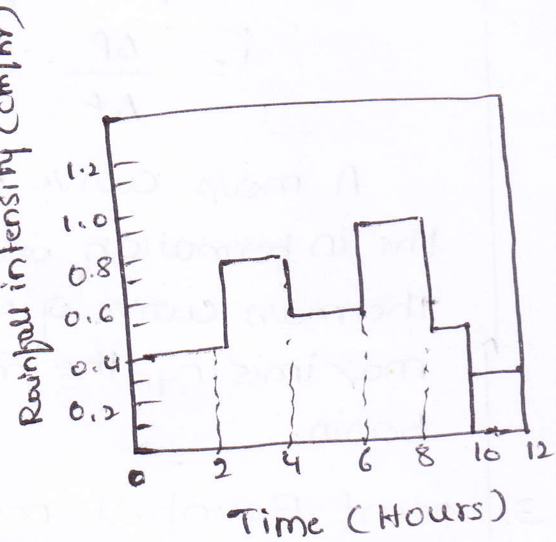
Presentation of Rainfall Data

There are three methods of presentation of rainfall data collected through measurement at a given station.

1. Hyetograph
2. Mass curve of rainfall method
3. point rainfall method.

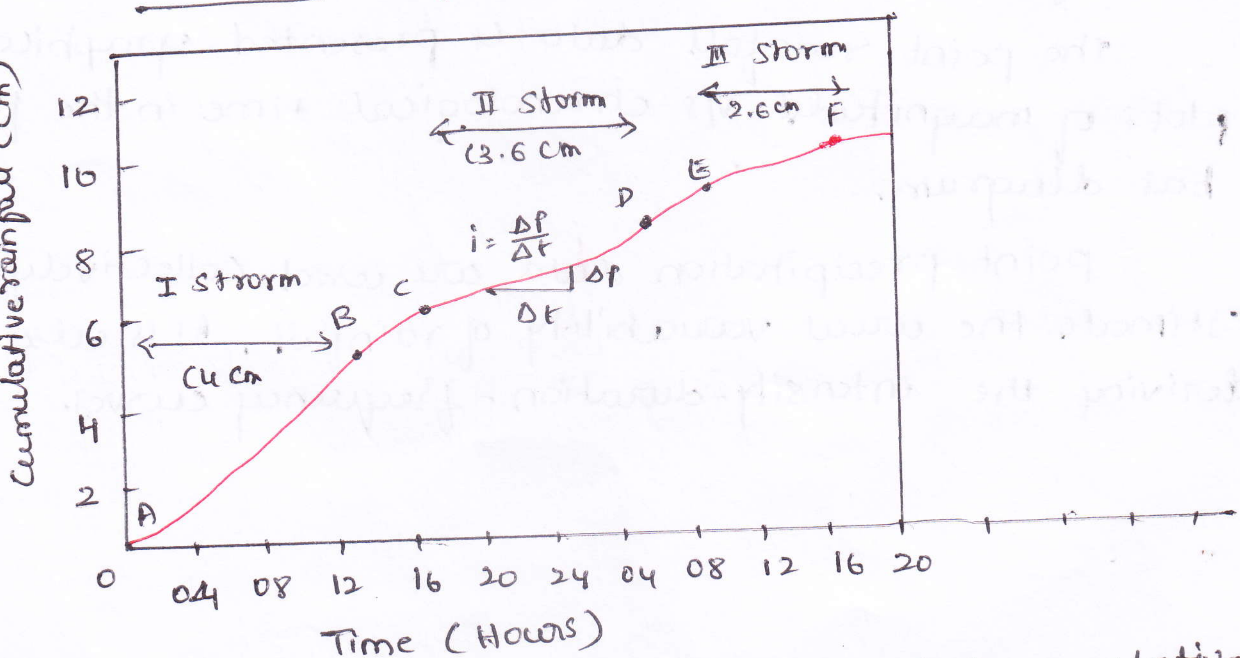
1. Heytograph Method

A hyetograph is a bar graph showing the intensity of rainfall with time as shown in fig.



The hyetograph can be prepared either from the mass curve of rainfall, or directly from the data obtained from automatic raingauges.

2. Mass curve of Rainfall



A mass curve of rainfall is a plot of cumulative depth of rainfall against time, as shown in fig.

The steepness of the curve indicates the intensity of rainfall. A horizontal portion of the curve indicates that there was no rainfall during that period.

Thus in fig. A to B is the first period of rainfall. There was no rainfall from B to C, & therefore the curve is horizontal C to D shows the second period of rainfall, while E to F shows third period of storm. The mass curve of rainfall is rising curve. The intensity of rainfall during any period is given by

$$i = \frac{\Delta P}{\Delta t}$$

A mass curve of rainfall is very usefull in Extracting the information on the duration & magnitude of a storm. The mass curve of a design storm can be obtained by maximising the mass curves of the severe storms in the basin.

3. point Rainfall method:

The rainfall data of a station is known as point rainfall of station rainfall. This data can be presented as daily, weekly, monthly, seasonal or annual values.

The point rainfall data is presented graphically as plots of magnitudes v/s chronological time in the form of a bar diagram.

point precipitation data are used collectively to estimate the areal variability of rainfall. It is also used in deriving the intensity-duration-frequency curves.

Estimation of missing Rainfall Data.

Prediction of the missing data can be made with the help of available data of nearby measuring stations.

1. Arithmetic mean method
2. Normal ratio method
3. Inverse distance method by U.S. weather Service.

1. Arithmetic mean method

According to this method, the missing rainfall P_x of the station x is computed by simple arithmetic average of the rainfall at the nearby stations (index station) in the following form:

$$P_x = \sum_{i=1}^{n} \frac{P_i}{n} = \frac{1}{n}(P_1 + P_2 + \dots P_n)$$

n = number of index stations

The above method is used only under the following conditions:

1) The normal annual rainfall of the missing station is within 10 % of the normal annual rainfall of the index stn.
2) Data of at least three index station should be available
3) The index stations should be evenly around the missing station & should be as close as possible.

2. Normal ratio method

In this method, the rainfall (P_i) of the surrounding index stations are weighed by the ratio of normal annual rainfall by using the following Equation

$$P_x = \frac{1}{n}\left[P_1 \frac{N_x}{N_1} + P_2 \frac{N_x}{N_2} + \dots P_n \frac{N_x}{N_n}\right]$$

$$= \frac{N_x}{n}\left[\frac{P_1}{N_1} + \frac{P_2}{N_2} + \dots \frac{P_n}{N_n}\right]$$

N_1, N_2, N = normal annual rainfall of index station.

N_x = normal annual rainfall of missing station

n = no. of index station.

3. Inverse distance method (U.S. weather service method)

In this method a set of rectangular co-ordinate axes are passed through the missing rain gauge station so that its co-ordinates are (0,0). The co-ordinates (x_i, y_i) of each index station, surrounding the missing station are found. The weightage (w_i) of each index station is represented by the inverse of the square of its distance from the missing station, & is given by.

$$w_i = \frac{1}{D_i^2} = \frac{1}{x_i^2 + y_i^2}$$

The missing rainfall data of the station x is then computed from the following equation.

$$P_x = \frac{\sum_{i=1}^{n} P_i w_i}{\sum_{i=1}^{n} w_i}$$

The above method gives good results & is therefore the most acceptable method for scientific analysis. However, the limitation of the method is that it estimates missing rainfall between the highest & the lowest values of the index station.

Moving Average curve:

The moving average curve smoothens out the extreme variations & indicate the trend of cyclic pattern, if any, more clearly. It is also known as the moving mean curve. The procedure to construct the moving average curve is as follows.

→ The moving average curve is constructed with a moving period of m year where m is generally taken to be 3 or 5 years.

→ Let $x_1, x_2, \ldots x_n$ be the sequence of given annual rainfall in the chronological order

→ Let y_i denote the ordinate of the moving average curve for the ith year. Then for $m=3$, y_i is computed from.

$$y_2 = \frac{x_1+x_2+x_3}{3}$$

$$y_3 = \frac{x_2+x_3+x_4}{3}$$

$$\vdots$$

$$y_i = \frac{x_{i-1}+x_i+x_{i+1}}{3}$$

$$\vdots$$

$$y_{n-1} = \frac{x_{n-2}+x_{n-1}+x_n}{3}$$

y value can be computed, the value of y correspond in time to the middle value of the x's being averaged & therefore it is convenient to use odd values of m.

A moving average of m applied to a sequence of n value yield a sequence of $(n-2K)$ values, $K = \frac{(m-1)}{2}$.

for any general m, the y terms can be expressed as

$$y_i = \frac{1}{m}\sum_{j=i-K}^{i+K} x_j \; ; \text{ for } i = K+1, K+2 \ldots (n-K)$$

Although it is possible to use moving averages with any m, it is necessary that m be small compared to n. The moving average technique can be applied to others hydrological parameters as well such of temperature, wind speed ect.

Problems

1) A watershed has four rain gauge stations, A, B, C & D. During a storm, rain gauge Station A was inoperative, while stations B, C, & D, surrounding station A, recorded rainfall of 48mm, 51mm & 45mm resp. Estimate the missing storm precipitation of station A, using AMM

Soln:

$$P_A = \frac{1}{3}(P_B + P_C + P_D) = \frac{1}{3}(48+51+45) = 48mm.$$

2) A precipitation station X was inoperative for some time during which a storm occurred. The storm totals at three stations A, B & C surrounding X, were respectively 6.60, 4.80 & 3.70 cm. The normal annual precipitation amounts at stations X, A, B & C are respectively 65.6, 72.6, 51.8 & 38.2 cm. Estimate the storm precipitation for station X.

Soln: If N_X, N_A, N_B & N_C are the average annual precipitation amounts at X, A, B & C & P_A, P_B & P_C are the storm totals of stations A, B & C surrounding X, the storm precipitation P at station X is given by.

$$P_X = \frac{1}{3}\left[\frac{N_X}{N_A} \times P_A + \frac{N_X}{N_B} \times P_B + \frac{N_X}{N_C} \times P_C\right]$$

$$P_X = \frac{1}{3}\left[\frac{65.6}{72.6} * 6.6 + \frac{65.6}{51.8} \times 4.8 + \frac{65.6}{38.2} \times 3.7\right] = \frac{1}{3}(18.33)$$

$$\boxed{P_X = 6.11 cm}$$

3) The recorded rainfall at 4 raingauge stations in an area for a period of 8 years is given below. Estimate the missing rainfall at station X in the year 1991

usf.

Year	Annual rainfall in cm at the 4 raingauge Station			
	X	A	B	C
1991	—	57.7	63.0	44.2
1992	121.0	108.3	100.9	95.1
1993	95.0	75.3	90.5	60.4
1994	69.0	67.5	58.6	62.3
1995	86.0	55.7	69.6	80.3
1996	59.9	52.3	75.4	85.1
1997	50.3	46.1	62.5	40.3
1998	34.8	47.6	52.0	30.1

Soln: Normal Annual rainfall at stations X, A, B & C

$$N_X = \frac{121.0 + 95 + \cdots + 50.3 + 34.8}{7} = \frac{516}{7} = 73.71 \text{ cm}$$

$$N_A = \frac{57.7 + 108.3 + \cdots + 46.1 + 47.6}{8} = \frac{510.5}{8} = 63.81 \text{ cm}$$

$N_B = 71.56$ cm $\quad N_C = 62.23$ cm.

$$P_x = \frac{1}{3}\left[P_A \times \frac{N_x}{N_A} + P_B \times \frac{N_x}{N_B} + P_c \frac{N_x}{N_c}\right]$$

$$= \frac{1}{3}\left[57.7 \times \frac{73.71}{63.81} + 63 \times \frac{73.71}{71.56} + 44.2 \times \frac{73.71}{62.23}\right]$$

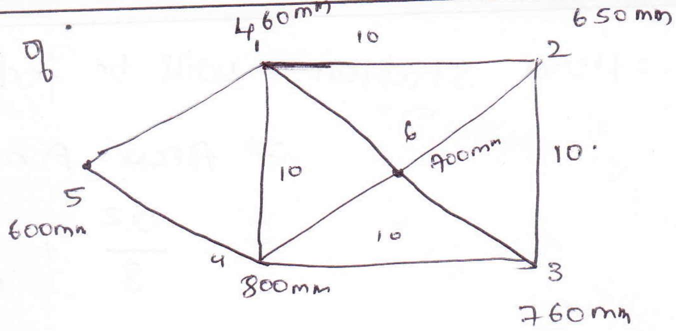
$$= \frac{1}{3}\left[66.65 + 64.89 + 52.35\right]$$

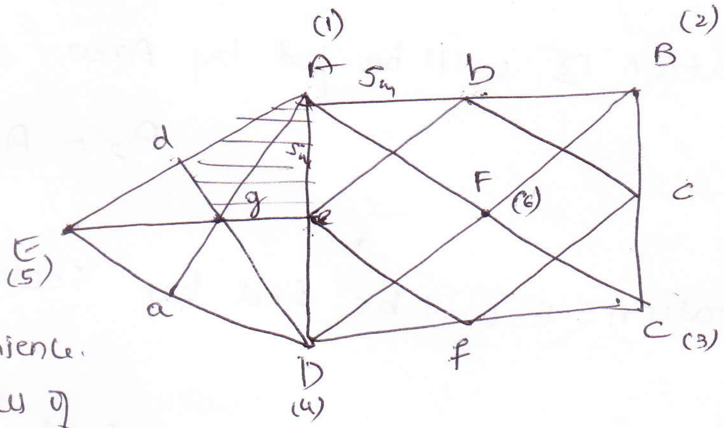
$$= \frac{183.89}{3}$$

$$\boxed{P_x = 61.3 \text{ cm}}$$

Q. P. Question paper problem

1) An area is composed of a square of side





Soln :

* Let the stations 1, 2, 3, 4, 5, 6 be named as stations ABCDE & F resp. for convenience.

Let the length of the sides of square ABCD is 10 km.

* Then the length of each side of the equilateral Δ^{le} plot will be also a.

* Now for the Δ^{le} plot, draw $\perp^r$ bisectors Aa, Dd, & Ee, so that they meet in point g. $\|^{ly}$, draw the perpendicular bisectors eb, bc, cf & fe of the lines FA, FB, FC & FD resp.

* Evidently, station F will be fed by the rectangular area, bcfe, where length of its side, say bc will be equal to $\frac{1}{2}AC = \frac{1}{2}\sqrt{2}a$

$$\text{Area bcfe} = A_6 = \frac{a}{\sqrt{2}} \times \frac{a}{\sqrt{2}} = \frac{a^2}{2} = \frac{(10)^2}{2} = 50 \text{ km}^2 \qquad AC = \sqrt{2}a$$

$$\Delta^{le} \text{ Abe} = \frac{1}{2} \times \text{be} \times \text{height} = \frac{1}{2} \times \frac{\sqrt{2}a}{2} \times \frac{\sqrt{2}a}{4} = \frac{2a^2}{16} = \frac{a^2}{8}$$

Sectional area $A_{dge} = \frac{1}{3}$ Area of Δ ADE

$$= \frac{1}{3} \times \frac{1}{2} \times a \times \frac{\sqrt{3}}{2} a = \frac{a^2}{4\sqrt{3}}$$

Hence station A will be fed by area

$= \text{Area Abe} + \text{area Adye}$

$= \frac{a^2}{8} + \frac{a^2}{4\sqrt{3}} = \frac{(10)^2}{8} + \frac{(10)^2}{4\sqrt{3}}$

$= 12.5 + 14.43 = 26.93 \text{ Km}^2$

Hence $A_1 = A_4 = 26.93 \text{ km}^2$

Station B will be fed by Area bcB = area Abe

$A_2 = A_3 = \frac{a^2}{8} = \frac{10^2}{8} = 12.5 \text{ Km}^2$

Station E will be fed by secti area Edga = Area Adge

$= \frac{a^2}{4\sqrt{3}}$

$A_5 = \frac{a^2}{4\sqrt{3}} = \frac{(10)^2}{4\sqrt{3}} = 14.43 \text{ Km}^2$

station F will be fed by area ebcf $= \left(\frac{a}{\sqrt{2}}\right)^2$

$A_6 = \frac{(10)^2}{2} = 50 \text{ Km}^2$

$$P_{av} = \frac{P_1 A_1 + P_2 A_2 + \dots + P_6 A_6}{A_1 + A_2 + \dots A_6}$$

$$= \frac{460 \times 26.93 + 650 \times 12.5 + 760 \times 12.5 + 800 \times 26.93 + 600 \times 14.43 + 700 \times 50}{26.93 + 12.5 + 12.5 + 26.93 + 14.43 + 50}$$

$= \frac{95214.8}{143.29} = 664.49 \text{mm}.$

$P_{av} = 664.49 \text{mm}$